Hardwick and Cambourne Calculation Methods and Progression 2023
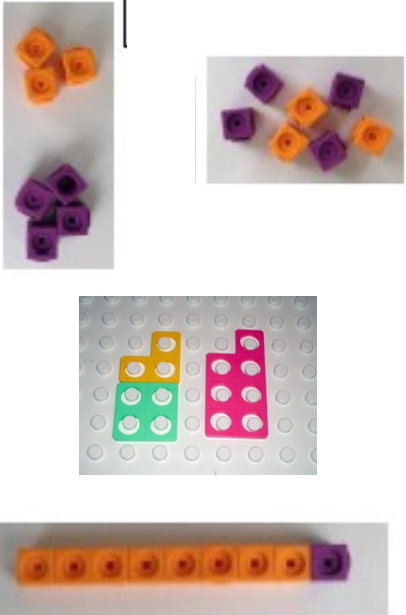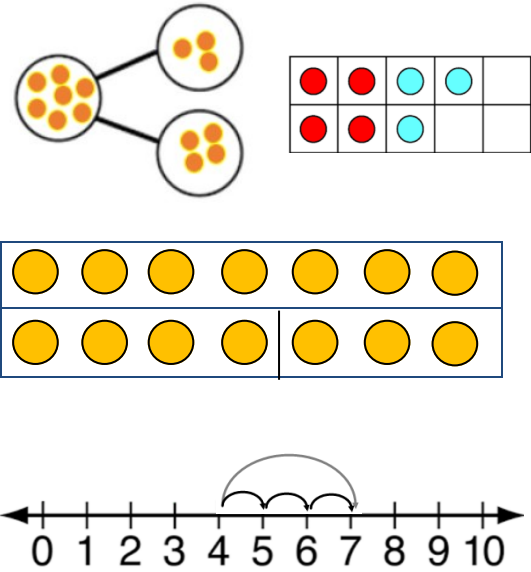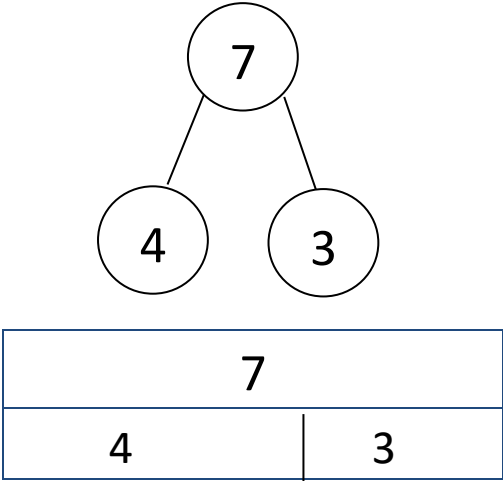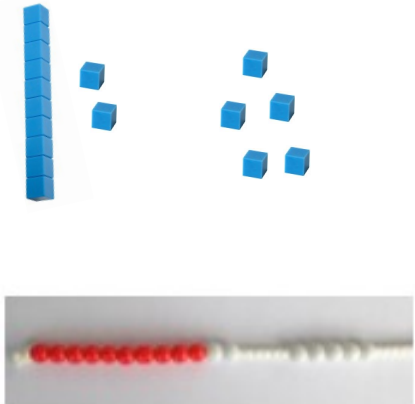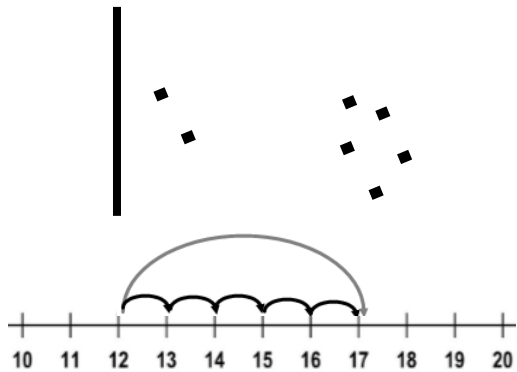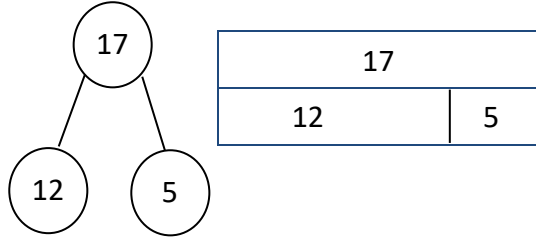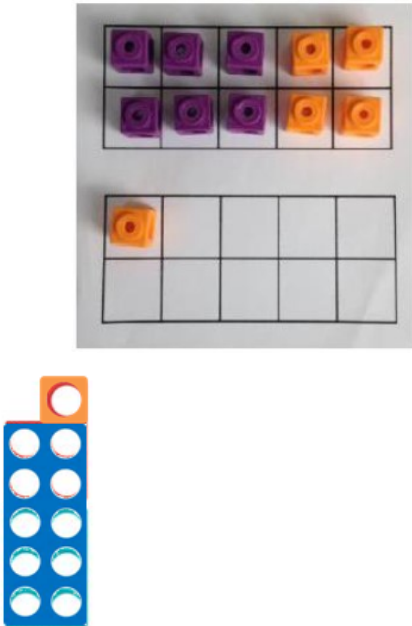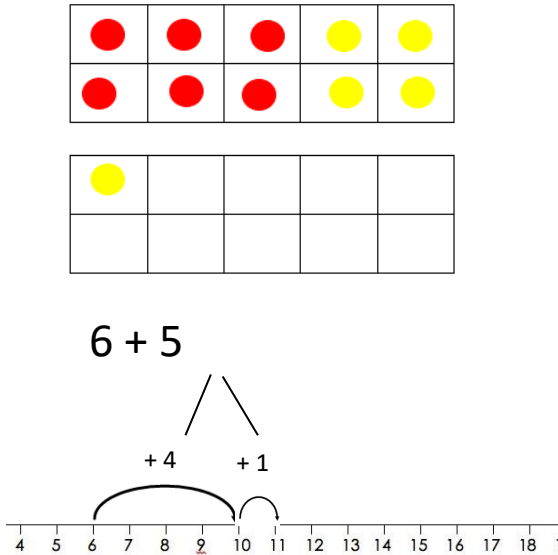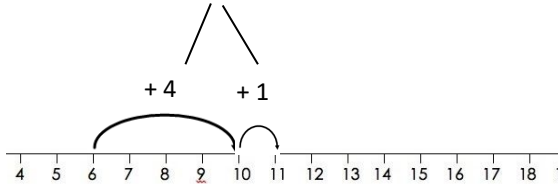
	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6
Addition	Combining two parts to make a whole: part whole model Starting at the bigger number and counting on Regrouping to make ten	Adding 3 single digits Bridging to ten Expanded column method then column method – 2 digit including regrouping	Column method – 3 digit including regrouping	Column method – 4digit including regrouping	Column method – regrouping (with more than 4 digits) Decimals with the same amount of decimal places	Column method –regrouping (up to millions) Decimals with the different amounts of decimal places
Subtraction	Taking away ones Counting back Find the difference Part whole model Bridging to 10	Counting back Find the difference Bridging to ten Expanded column method – 2 digit no regrouping- moving on the regrouping (alongside number line)	Column method – 3 digit including regrouping	Column method – 4 digit including regrouping	Column method – regrouping (with more than 4 digits) Decimals with the same amount of decimal places	Column method – regrouping (up to millions) Decimals with the different amounts of decimal places
Multiplication	Doubling Counting in multiples Arrays Repeated addition	Doubling Counting in multiples Arrays showing commutative multiplication Repeated addition	Counting in multiples Repeated addition Arrays – showing commutative multiplication Grid method/partitioning method	Column multiplication (2 and 3 digit multiplied by 1 digit)	Column multiplication (up to 4 digit numbers multiplied by 1 or 2 digits)	Column multiplication (3 digit x 3 digit and decimals) (multi digit up to 4 or 5 digits by a 2 digit number)
Division	Sharing objects into groups Division as grouping Halving	Division as grouping Division within arrays Halving	Division within arrays Division with a remainder Division using x table facts Short division (2 digit by 1 digit – concrete and pictorial)	Division using x table facts Short division (up to 3 digits)	Short division (up to 4 digits and decimals by a 1 digit number, interpret remainders appropriately for the context)	Short division (up to 4 digits by a 1 digit number or 2 digits up to 12 and decimals) Long division (up to 4 digits by a 2 digit number – interpret remainders as whole numbers, fractions decimals or round)

Calculation Methods and Progression
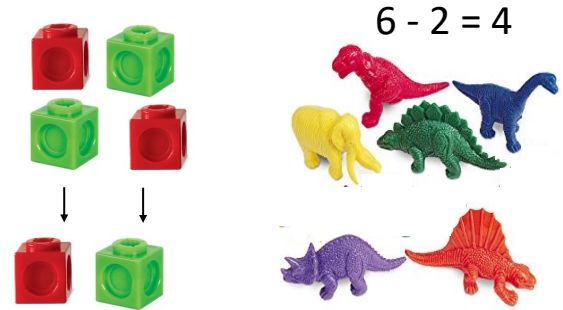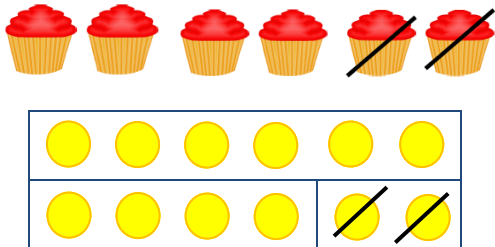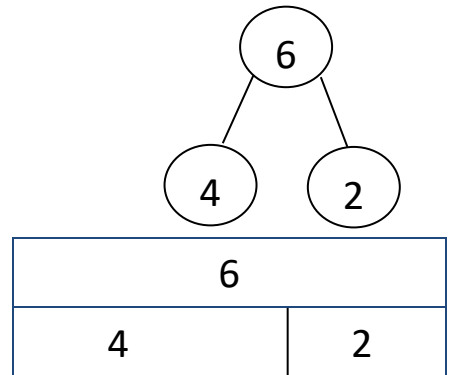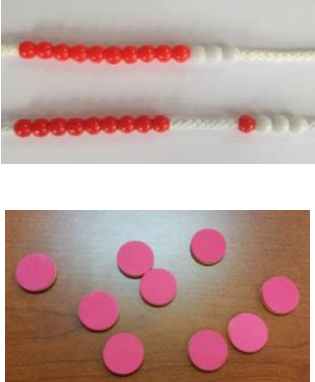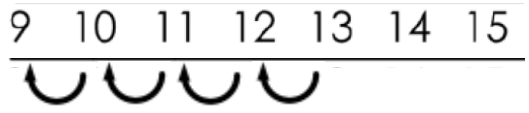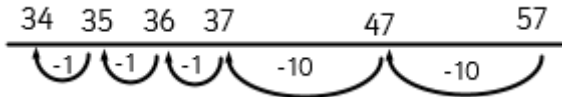
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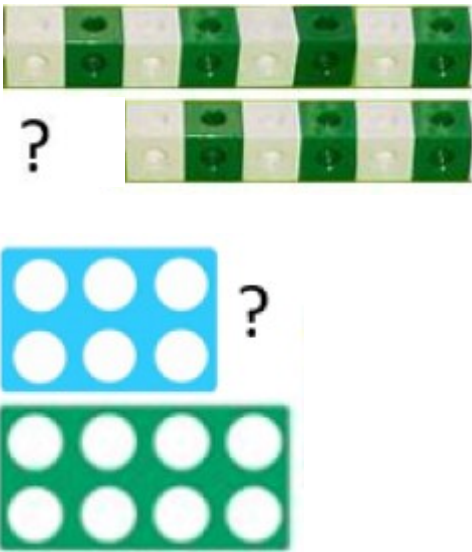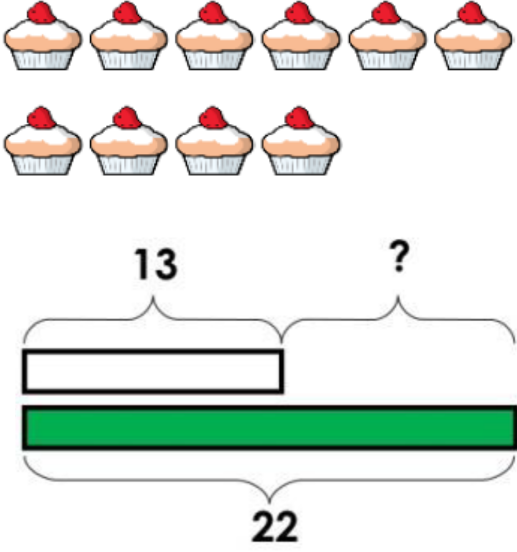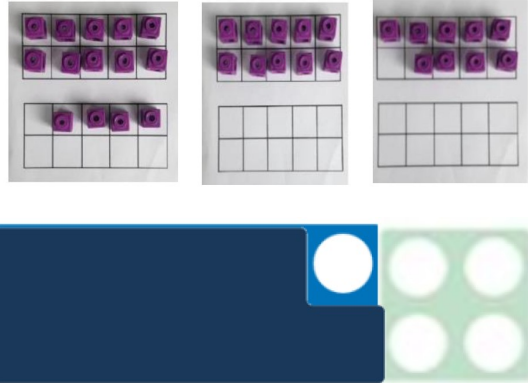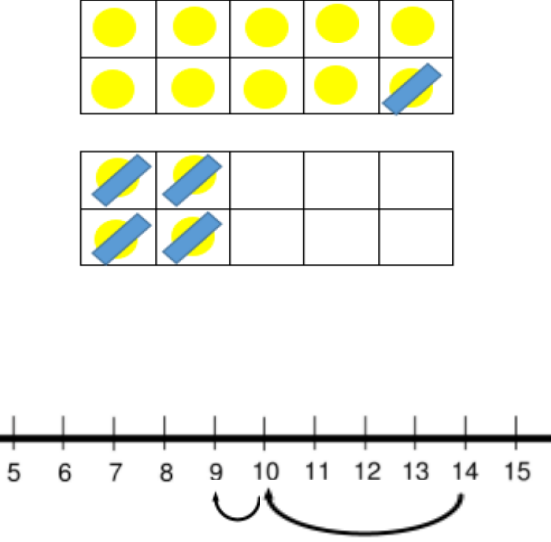
	Concrete	Pictorial	Abstract
Adding 1 digit numbers (combining two parts to make a whole)			$4 + 3 = 7$
Counting on (numbers to 20)			$12 + 5 = 17$ Place larger number in your head and count on the smaller number to find your answer
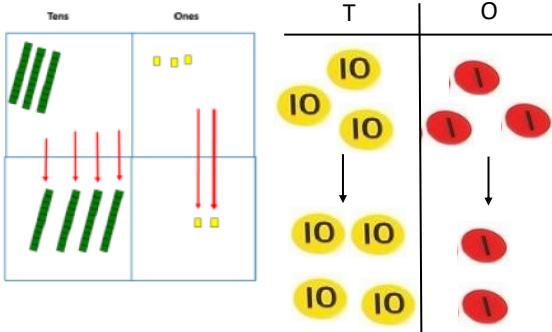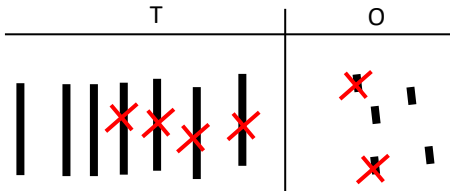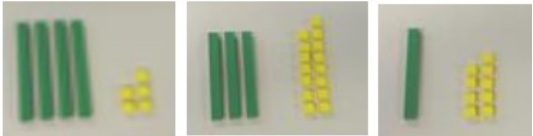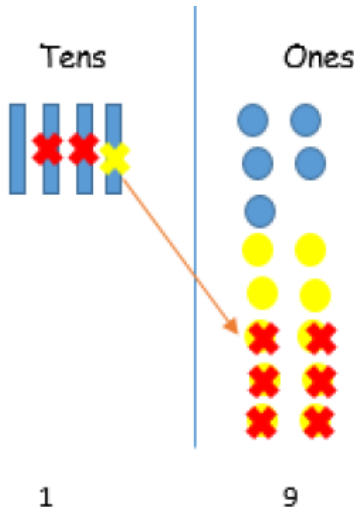
	Concrete	Pictorial	Abstract
Bridging to 10		 6 + 5 	6 + 5 = 11 6 + 4 + 1 = 11

Column method without re-grouping			30 + 4
			+
			20 + 2
			―――――
			50 + 6 = 56
			
			Moving on to:
			
			  3 4 51.23
			+ +
			  2 2 25.64
			――― ―――
			  5 6 76.87
			――― ―――
			
			(Apply to larger numbers and decimals. Ensure decimal point is put under before calculating)

	Concrete	Pictorial	Abstract
Column method with re-grouping			$30 + 6$ $+$ $20 + 5$ ――――― $60 + 1$ $= 61$ $\quad$ 10 Moving on to: $\quad 3\ 6$ $+$ $\quad 2\ 5$ ――――― $\quad 6\ 1$ ――――― $\quad$ 1 Also apply to larger numbers and decimals or addition of more than two numbers e.g. $\quad 2\ 3\ 1\ .\ 5$ $+\ 1\ 7\ 4\ .\ 2$ ――――――― $\quad 2\ 0\ 5\ .\ 7$ ――――――― $\quad$ 1 $\quad\quad 1\ 3\ 5\ .\ 5$ $\quad\quad 2\ 7\ 3\ .\ 8$ $+\ 4\ 6\ 9$ ――――――― $\quad\quad 8\ 7\ 8\ .\ 3$ ――――――― $\quad\quad$ 1 1 1

Subtraction
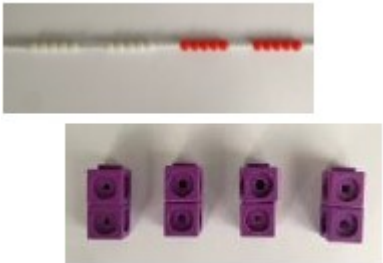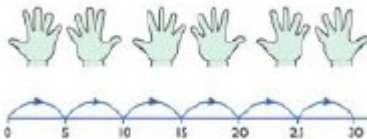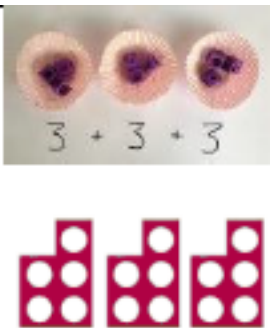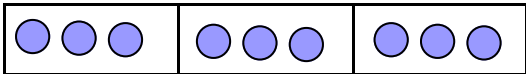
	Concrete	Pictorial	Abstract
Taking away ones	Use physical objects to show how they can be taken away. 6 - 2 = 4	Cross out drawn objects to show what has been taken away	6 - 4 = 2
Counting back	Move the beads/counters away from the group counting back-wards as you go	13 - 4 = 9 9 10 11 12 13 14 15 Progress to two digit numbers, jumping in tens then ones 57 - 23 = 34 34 35 36 37 47 57 -1 -1 -1 -10 -10	13 - 4 = 9 Put 13 in your head, count back 4. What number are you at? Progress to bridging to ten e.g. 13 - 3 - 1
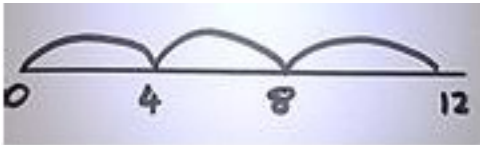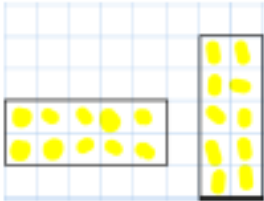
	Concrete	Pictorial	Abstract
Find the difference/ How many more?		13 ? 22	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches by counting on.
Bridging to 10	14 - 5 Take away 4 then take away one more 	 5 6 7 8 9 10 11 12 13 14 15	14 - 5= 14 - 5 = 9 4 1 5 is made up of a 4 and a 1 so I can subtract 4 to make 10, then1 to get to 9.
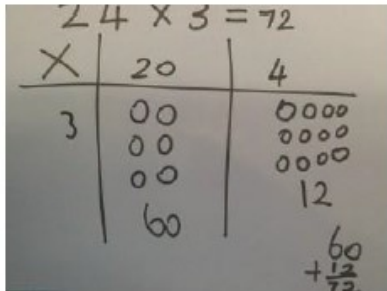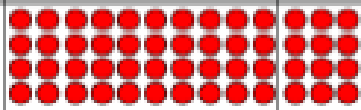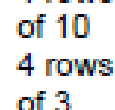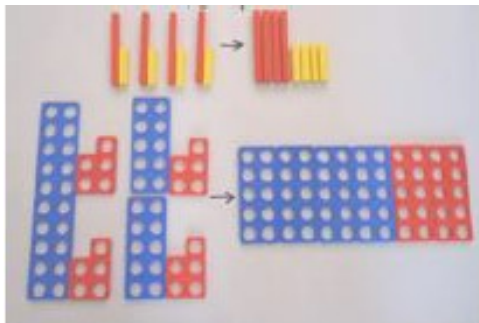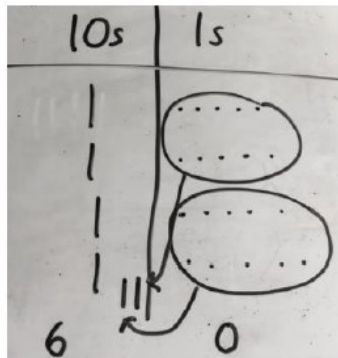
	Concrete	Pictorial	Abstract
Column method without regrouping	75 - 42 = 33		75 - 42= $\begin{array}{r} 70 + 5 \\ - 40 + 2 \\ \hline 30 + 3 = 33 \end{array}$ Expanded column method Moving on to formal column method: $\begin{array}{r} 75 \\ - 42 \\ \hline 33 \end{array}$ or $\begin{array}{r} 8296.5 \\ - 5174.2 \\ \hline 3122.3 \end{array}$
Column method with regrouping	45 - 26 = 19 1) Start by partitioning 45 2) Exchange one ten for ten more ones 3) Subtract the ones, then the tens.		45 - 26 = $\begin{array}{r} \overset{30}{\cancel{40}} + \overset{1}{}5 \\ - 20 + 6 \\ \hline 10 + 9 = 19 \end{array}$ Moving on to formal column method: $\begin{array}{r} \overset{3}{\cancel{4}}\,\overset{1}{5} \\ - 26 \\ \hline 19 \end{array}$ or $\begin{array}{r} \overset{7}{\cancel{8}}2\,\overset{8}{\cancel{9}}.\overset{1}{1}5 \\ - 537.6 \\ \hline 291.55 \end{array}$
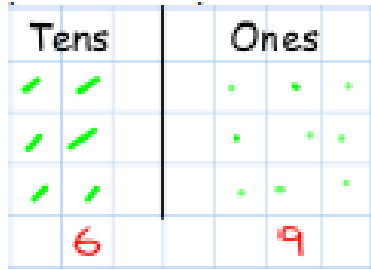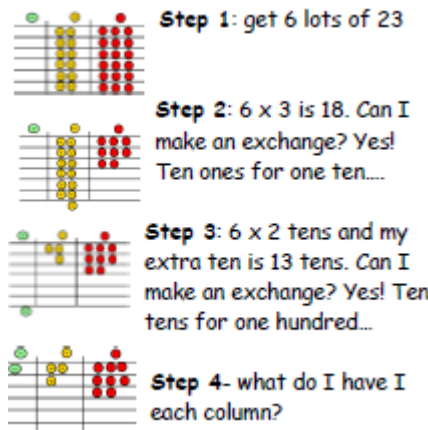
Multiplication

	Concrete	Pictorial	Abstract
Counting in multiples	Count in multiples supported by concrete objects in equal groups.	Using a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 0, 2, 4, 6, 8 0, 5, 10, 15, 20
Repeated grouping/ repeated addition	Use different object to add equal groups.	Children to represent the practical resources in a picture e.g. Use the bar model for a more structured visual.	Write addition sentences to describe objects and pictures. 3 + 3 + 3 = 9
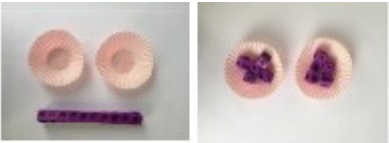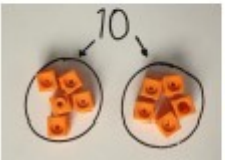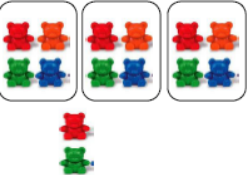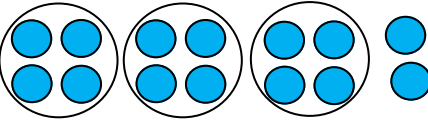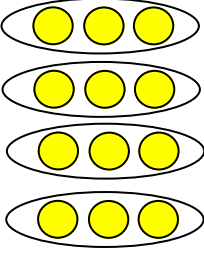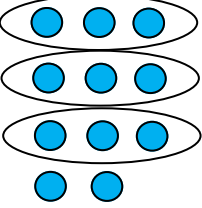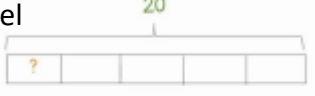
	Concrete	Pictorial	Abstract
Use number lines to show repeated groups		Represent this pictorially alongside a number line 	Abstract number line **3 x 4 = 12**
Use arrays to illustrate commutativity	Create arrays using multiplication counters/cubes/ Cuisenaire rods to show multi-plication sentences **2 x 5 = 5 x 2** 	Children to draw the arrays 	Children to be able to use an array to write a range of calculations e.g. 2 x 5 = 10 5 x 2 = 10 2 +2 + 2 + 2 + 2 = 10 5 + 5 = 10

	Concrete	Pictorial	Abstract
Grid method	Show the link with arrays to first introduce the grid method. 4 rows of 10 4 rows of 3	Children can represent the work they have done with place value counters or dienes. 	Children to start with multiplying by one digit numbers and showing the clear addition alongside the grid. 210 + 35 = 245
Partition to multiply	Use numicon, dienes, Cuisenaire rods) 15 x 4 	Children to represent the concrete objects pictorially. 	Children to be encouraged to show the steps they have taken 15 x 4 10 5 10 x 4 = 40 5 x 4 = 20 40 + 20 = 60

	Concrete	**Pictorial**	**Abstract**
Formal column method (no exchanging)	Children to use place value counters or dienes (at the first stage—no exchanging) 23 x 3 Make 23, 3 times. See how many ones and then how many tens	Children to represent the counters in a pictorial way 	Children to record what they are doing to show understanding 23 x 3 23 (moving on to compact method) x 3 ——— 9 (3 x 3) 60 (3 x 20) ——— 69 23 x 3 ——— 69 ———
Formal column method (exchanging)	Children to use place value counters/dienes to understand how the column method works 23 x 6 Step 1: get 6 lots of 23 Step 2: 6 x 3 is 18. Can I make an exchange? Yes! Ten ones for one ten…. Step 3: 6 x 2 tens and my extra ten is 13 tens. Can I make an exchange? Yes! Ten tens for one hundred… Step 4- what do I have I each column?	Children to represent the counters/dienes pictorially 	Children to record the calculations they are making 23 x 6 = (moving on to compact method) 23 x 6 ——— 18 (6 x 3) 120 (6 x 20) ——— 138 (Including decimals multiplied by whole numbers) 23 x 6 ——— 138 1 1 2 3 1 . 5 X 4 ———————— 9 2 6 . 0 1 2

Concrete	Pictorial	Abstract

When children start to multiply 3-digit by 3-digit and 4 digit by 2 digit etc, they should be confident with the abstract and compact methods

124 x 26

```
    1  2  4
 ×     2  6
 ─────────────
    ₋7  4  4
     ₁  ₂
  2 ₋4  8  0
 ─────────────
  3  2  2  4
  1  1
```

Answer: 3224

This method could be used for struggling learners:

```
   124          124          2480
 x   6        x  20        +  744
 ──────       ──────        ──────
   744         2480          3224
  1  2                       1  1
```

Division

	Concrete	Pictorial	Abstract
Division as sharing	Children use concrete objects to share equally With remainders e.g. 14 ÷ 3 =	Children can use pictures or shapes to share quantities. This can also be done in a bar so all 4 operations have a similar structure: With remainders e.g. 14 ÷ 3 =	6 ÷ 2 = 3 What's the calculation?
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding. 10 ÷ 2 = 5 With remainders 13 ÷ 4 = 3 r 1	Draw arrays to see how many groups you can make e.g. 12 ÷ 3 = 4 With remainders 11 ÷ 3 = 3 r 2	Bar model 20 ÷ 5 = ? 5 x ? = 20 28 ÷ 7 = 4 How any groups of seven are there in 28? Count forward mentally in multiples and then see how many more are needed to find a remainder.

	Concrete	Pictorial	Abstract
Linking division to multiplication	Link division to multiplication by creating an array and thinking about the number sentences that can be created. E.g. $15 \div 3 = 5$ $5 \times 3 = 15$ $15 \div 5 = 3$ $3 \times 5 = 15$	Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division by creating four linking number sentences. E.g. $3 \times 5 = 15$ $5 \times 3 = 15$ $15 \div 5 = 3$ $15 \div 3 = 5$

| Short division (the bus stop method) | Use place value counters to group. 1. Make 615 with place value counters. 2. How many groups of 5 hundreds can you make with 6 hundred counters? 3. Exchange 1 hundred for 10 tens. 4. How many groups of 5 tens can you make with 11 ten counters? 5. Exchange 1 ten for 10 ones. 6. How many groups of 5 ones can you make with 15 ones? | Children can represent the place value counters pictorially or draw diagrams. This can then support them with moving onto the bus stop method. | Children can then move onto short division (bus stop method). $$5 \overline{)6^{1}1^{1}5}$$ 123 Begin with divisions that divide equally with no remainder. $$4 \overline{)8\ 7^{3}2}$$ 2 1 8 Move onto divisions with a remainder. $$5 \overline{)4\ 3^{3}2}$$ 8 6 r 2 Finally move onto decimal places. $$5 \overline{)4\ 3^{3}2\ .\ ^{2}0}$$ 8 6 . 4 Or express as a fraction $86 \frac{2}{5}$ |
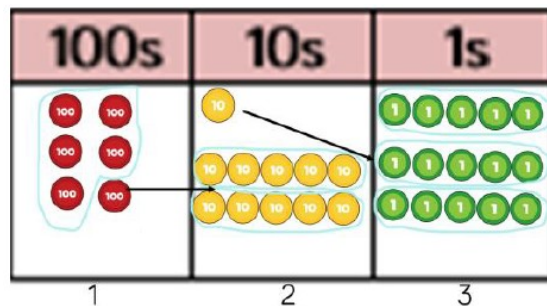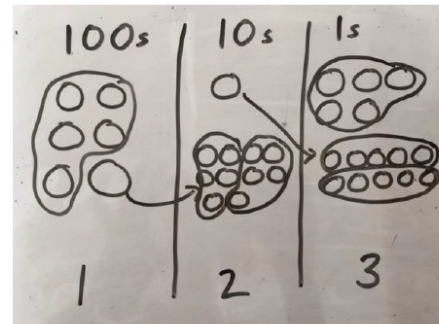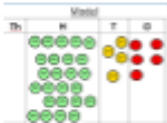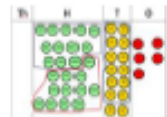
	Concrete	Pictorial	Abstract
Long division (for dividing by 2 digit numbers only)	$2544 \div 12$ How many groups of 12 thousands do we have? None Exchange 2 thousand for 20 hundreds. How many groups of 12 are in 25 hundreds? 2 groups. Circle them. We have grouped 24 hundreds so can take them off and we are left with one. Exchange the one hundred for ten tens so now we have 14 tens. How many groups of 12 are in 14? 1 remainder 2. Exchange the two tens for twenty ones so now we have 24 ones. How many groups of 12 are in 24? 2	Children can continue to represent the counters pictorially	Start with numbers that divide with no remainder $$\begin{array}{r} 2\ 7 \\ 15\ \overline{)4\ 0\ 5} \\ 3\ 0 \\ \hline 1\ 0\ 5 \\ 1\ 0\ 5 \\ \hline 0 \end{array}$$ (Move to answers with remainders) $$\begin{array}{r} 2\ 8\ r\ 12 \\ 15\ \overline{)4\ 3\ 2} \\ 3\ 0 \\ \hline 1\ 3\ 2 \\ 1\ 2\ 0 \\ \hline 1\ 2 \end{array}$$ (Also express as fractions: $28\frac{12}{15} = 28\frac{4}{5}$ Move onto decimal answers) $$\begin{array}{r} 2\ 8\ .\ 8 \\ 15\ \overline{)4\ 3\ 2\ .\ 0} \\ 3\ 0 \\ \hline 1\ 3\ 2 \\ 1\ 2\ 0 \\ \hline 1\ 2\ 0 \\ 1\ 2\ 0 \\ \hline 0 \end{array}$$ Children should list the multiples first e.g. 1 x 15 = 15 2 x 15 = 30 3 x 15 = 45 4 x 15 = 60 5 x 15 = 75 6 x 15 = 90 7 x 15 = 105